Recognizing Reduplicated Forms: Finite-State Buffered Machines

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The puzzle of reduplication I

- Copying in natural language phonology and morphology

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  - Total reduplication: Dyirbal plurals (Dixon, 1972, 242):

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gulgiɽi	*'prettily painted men'*	gulgiɽi-gulgiɽi	*'lots of prettily painted men'*

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  - Partial reduplication: Agta plurals (Healey, 1960,7):

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- Reduplication is common cross-linguistically.
  - 313 out of 368: productive reduplication ✓[1]
- String reversals are rarely attested.
  - confined to language games (Bagemihl, 1989)

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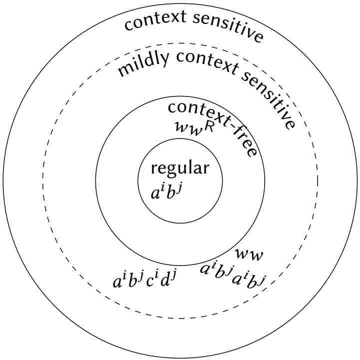
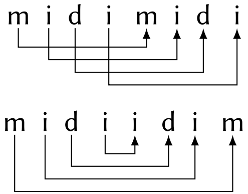
The puzzle of reduplication II





- unbounded copying
  - CH: mildly context-sensitive
  - NL: prevalent
- string reversal
  - CH: context-free
  - NL: rare
- most phonology and morphology: regular

The puzzle of reduplication

How can one fit in reduplicated strings without including some unattested context-free patterns, such as reversals?

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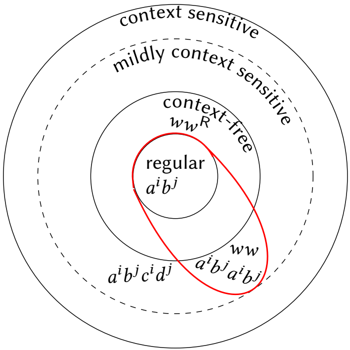
Gazdar and Pullum (1985, p. 278)

> *We do not know whether there exists an independent characterization of the class of languages that includes the regular sets and languages derivable from them through reduplication,...this class might be relevant to the characterization of NL word-sets.*

Goals of this talk [2]



- give a formal characterization of regular languages + the copying-derived languages
  - unbounded copying ✓
  - string reversal ✗
  - Swiss-German crossing dependencies ✗

[2] Due to time limit, previous approaches on computing reduplication, such as finite-state registered machines as recognizers for *bounded copying* (Cohen-Sygal and Wintner, 2006), 2-way finite-state transducers and further sub-classes (Dolatian and Heinz, 2018a; Dolatian and Heinz, 2018b; Dolatian and Heinz, 2019; Dolatian and Heinz, 2020) that model reduplication as *functions*, and the comparison can be found in the whole paper.

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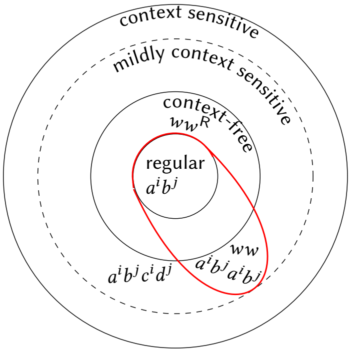

- give a formal characterization of regular languages + the copying-derived languages
  - unbounded copying ✓
  - string reversal ✗
  - Swiss-German crossing dependencies ✗
- analyze the closure properties of this class of languages

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Roadmap

1. Finite-state buffered machines
   - Intuitions and definitions
   - Examples
   - Different state arrangements
2. Some closure properties of FSBM Languages
   - Intersection with regular languages
   - Regular operations
   - Homomorphism and inverse homomorphism
3. Discussion and conclusion
   - Typology of reduplication

Outline

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Finite-state buffered machines

Finite-state automata + a copying mechanism

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- The copying mechanism
  - an unbounded memory buffer, with queue storage

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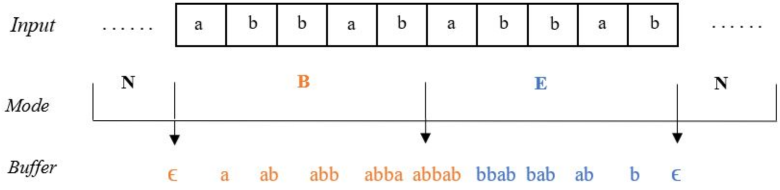
Finite-state automata + a copying mechanism

- The copying mechanism
  - an unbounded memory buffer, with queue storage
  - three different modes
    - normal (N) mode: similar to a normal FSA
    - buffering (B) mode: copying input symbols to the buffer
    - emptying (E) mode: matching symbols in memory against the input



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Based on different states...

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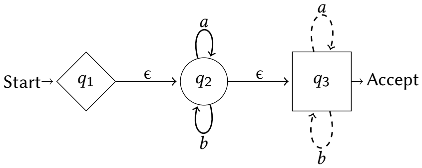
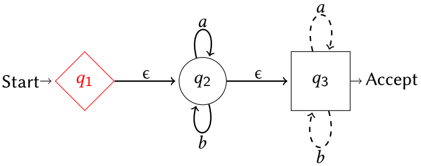


Figure: $L(M_1) = \{ww|w \in \{a,b\}^*\}$

When to switch to which mode?

Based on different states...



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- q_1: "switch to buffering mode, please"

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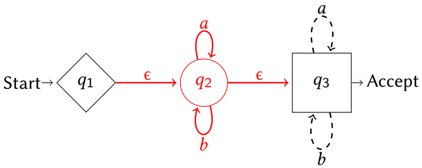

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- q_1: "switch to buffering mode, please"
- keeps in buffering mode and stores symbols in the buffer (queue)

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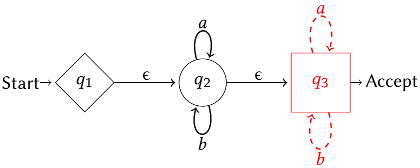

Figure: $L(\mathsf{M}_1) = \{ww | w \in \{a, b\}^*\}$

- q_1: "switch to buffering mode, please"
- keeps in buffering mode and storing symbols in the buffer (queue)
- q_3: "stop buffering and let's empty the buffer if strings match up "

Finite-state buffered machines: formal definition

A **Finite-State Buffered Machine (FSBM)** is a 7-tuple $\langle \Sigma, Q, I, F, \mathbf{G}, \mathbf{H}, \delta \rangle$ where

- Q: a finite set of states
- $I \subseteq Q$: initial states
- $F \subseteq Q$: final states
- $G \subseteq Q$: **states where the machine must enter buffering (B) mode**
- $H \subseteq Q$: **states visited while the machine is emptying the buffer**
- $G \cap H = \varnothing$
- δ: $Q \times (\Sigma \cup \{\epsilon\}) \times Q$: the state transitions according to a specific symbol

Finite-state buffered machines: configuration

A configuration of an FSBM $D = (u, q, \mathbf{v}, \mathbf{t}) \in \Sigma^* \times Q \times \Sigma^* \times \{\text{N}, \text{B}, \text{E}\}$

- u: the input string
- q: the state the machine is currently in
- v: **the string in the buffer**
- t: **the mode the machine is currently in**

Finite-state buffered machines: configuration transition

Given an FSBM M and $x \in (\Sigma \cup \{\epsilon\})$, $u, w, v \in \Sigma^*$, we define a configuration D_1 **yields** a configuration D_2 in M ($D_1 \vdash_M D_2$) as the smallest relation such that:

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- For every transition (q_1, x, q_2) with at least one state of $q_1, q_2 \notin H$
  $(xu, q_1, \epsilon, N) \vdash_M (u, q_2, \epsilon, N)$ with $q_1 \notin G$ "normal" actions
  $(xu, q_1, v, B) \vdash_M (u, q_2, vx, B)$ with $q_2 \notin G$ "buffering" actions
- For every transition (q_1, x, q_2) and $q_1, q_2 \in H$
  $(xu, q_1, xv, E) \vdash_M (u, q_2, v, E)$ "emptying" actions

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- For every transition (q_1, x, q_2) and $q_1, q_2 \in H$
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- For every $q \in G$
  $(u, q, \epsilon, \text{N}) \vdash_M (u, q, \epsilon, \text{B})$ — *mode-changing actions*
- For every $q \in H$
  $(u, q, v, \text{B}) \vdash_M (u, q, v, \text{E})$ — *mode-changing actions*
  $(u, q, \epsilon, \text{E}) \vdash_M (u, q, \epsilon, \text{N})$ — *mode-changing actions*

An example run: abbabb

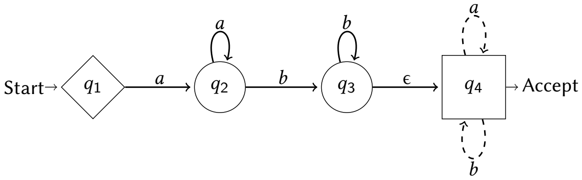

Figure: *An FSBM* M_2 *with* $G = \{q_1\}$ *and* $H = \{q_4\}$. $L(\mathsf{M}_2) = \{a^i b^j a^i b^j \mid i, j \geq 1\}$

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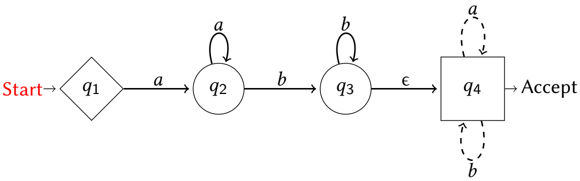

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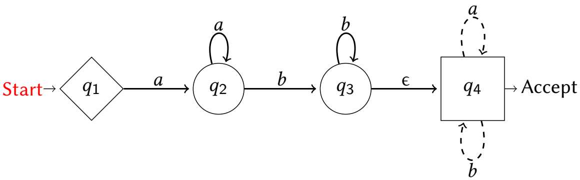

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input	state	buffer	mode
abbabb	q_1	ϵ	N

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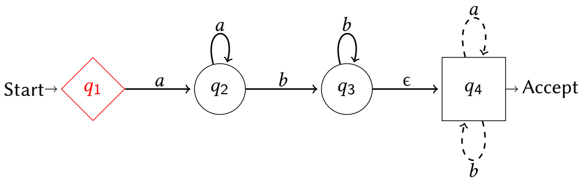

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input	state	buffer	mode
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abbabb	q_1	ϵ	B

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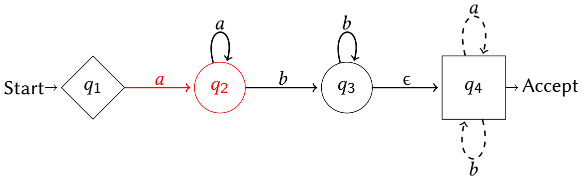

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abbabb	q_1	ϵ	B
bbabb	q_2	a	B

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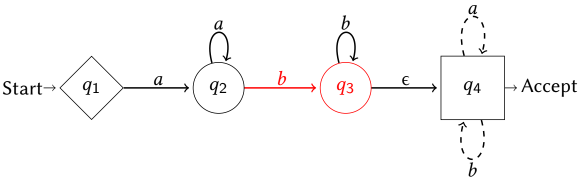

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bbabb	q_2	a	B
babb	q_3	ab	B

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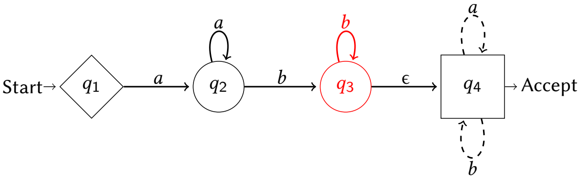

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bbabb	q_2	a	B
babb	q_3	ab	B
abb	q_3	abb	B

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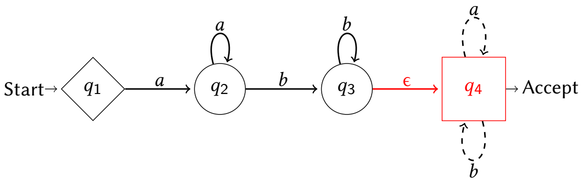

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abb	q_4	abb	B

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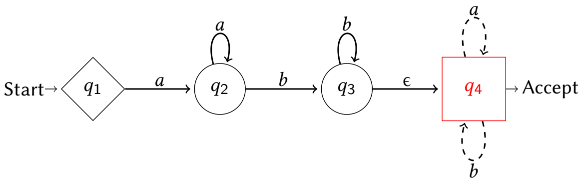

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input	state	buffer	mode
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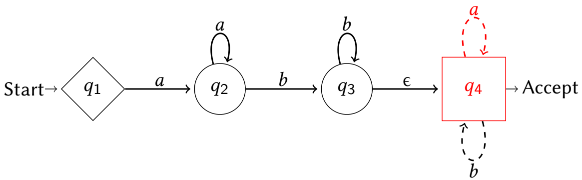

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abb	q_4	abb	B

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abb	q_4	abb	E
bb	q_4	bb	E

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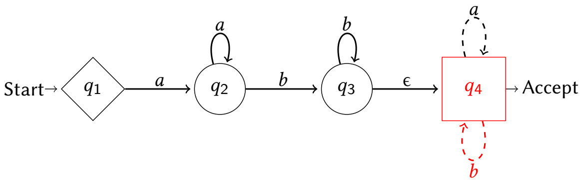

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abb	q_3	abb	B
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input	state	buffer	mode
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bb	q_4	bb	E
b	q_4	b	E

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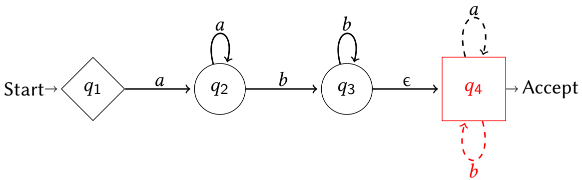

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abb	q_3	abb	B
abb	q_4	abb	B

input	state	buffer	mode
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b	q_4	b	E
ϵ	q_4	ϵ	E

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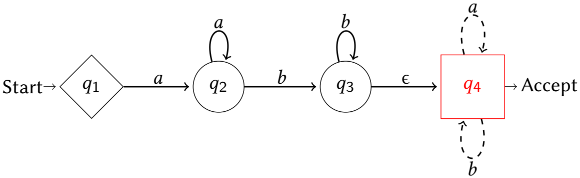

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abb	q_3	abb	B
abb	q_4	abb	B

input	state	buffer	mode
abb	q_4	abb	E
bb	q_4	bb	E
b	q_4	b	E
ϵ	q_4	ϵ	E
ϵ	q_4	ϵ	N

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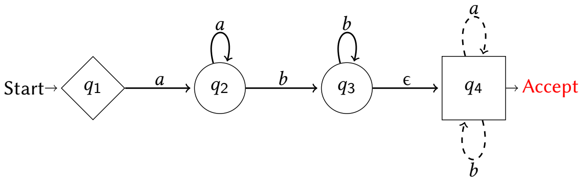

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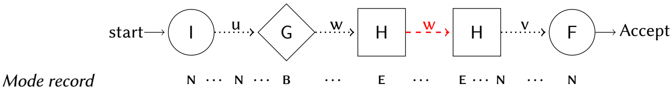
babb	q_3	ab	B
abb	q_3	abb	B
abb	q_4	abb	B

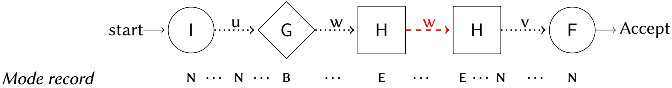
input	state	buffer	mode
abb	q_4	abb	E
bb	q_4	bb	E
b	q_4	b	E
ϵ	q_4	ϵ	E
ϵ	q_4	ϵ	N
ACCEPT			

Different arrangements of G & H states

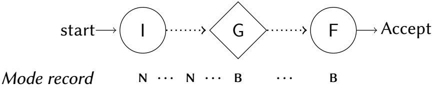


(a) *Useful arrangements of G and H states*

Different arrangements of G & H states

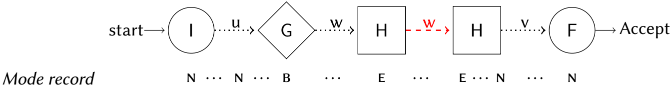


(a) *Useful arrangements of G and H states*



(b) *Non-Useful arrangements: No H states*

Different arrangements of G & H states



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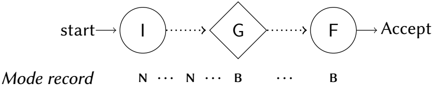

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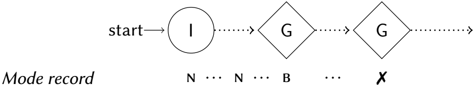

(c) *Non-Useful arrangements: No H states in between two G states along a path*

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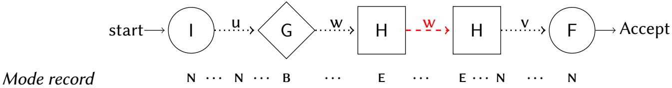

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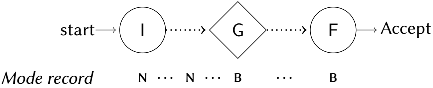

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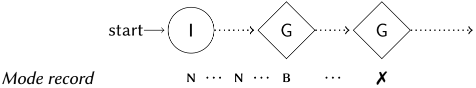

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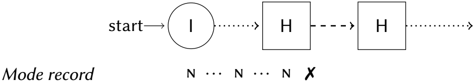

(d) *Non-Useful arrangements: H states before G states*

Outline

1. Finite-state buffered machines
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   - Regular operations
   - Homomorphism and inverse homomorphism
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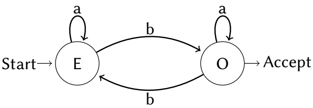
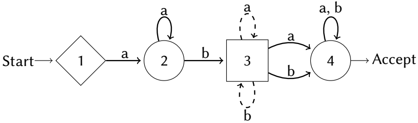
Intersection with regular languages

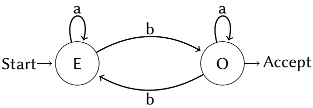


(a) *an FSA enforcing odd number of bs in a string. State E and State O represent even, odd number of bs in the prefix respectively*



(b) *a machine recognizing initial 'aa* b'-identity.* G = {1}, H = {3}

Intersection with regular languages



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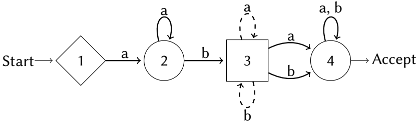

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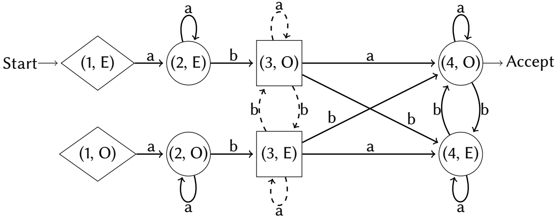
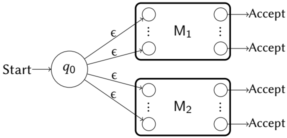


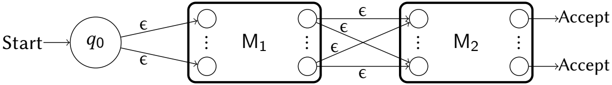
(c) *the intersection FSBM after the construction:* G = {(1, E), (1, O)}, H = {(3, 0), (3, E)}.

Regular operations [3]

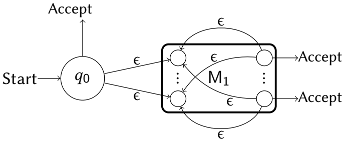
- union



- concatenation:

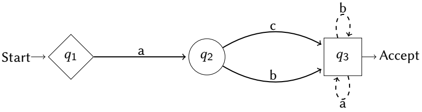


- Kleene Star



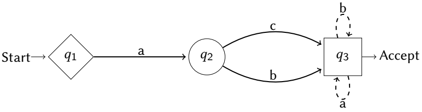
[3] That FSBM languages are closed under regular operations hints that the set of languages recognized by the new automata might be equivalent to the set of languages denoted by a version of regular expression with copying added.

Homomorphism



(a) $\mathsf{L}(\mathsf{M}) = \{abab\}$

Homomorphism



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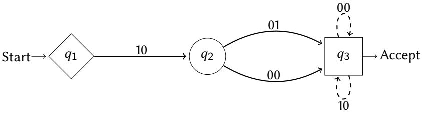
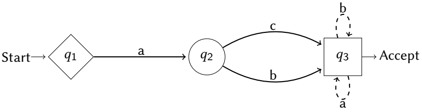


(b) $h(a) = 10, h(b) = 00, h(c) = 01$. *The intermediate step when the arcs are relabeled with mapped strings*

Homomorphism



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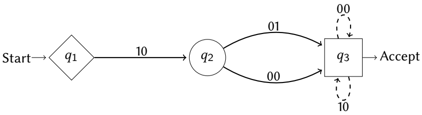

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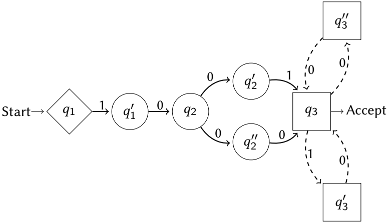
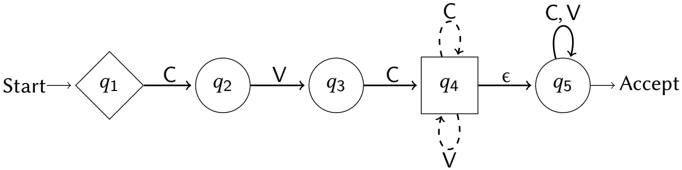
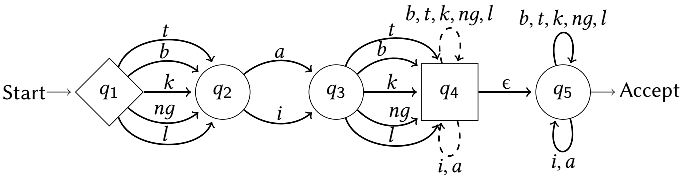


(c) *States* $q_1', q_2', q_2'', q_3', q_3''$ *are added to split the arcs.* $\mathsf{L}(\mathsf{M}_h) = \{10001000\}$

Inverse homomorphism?



(a) *An FSBM on the alphabet* $\{\mathsf{C}, \mathsf{V}\}$*: Agta initial CVC-reduplication. Assume* $\Sigma = \{t, b, k, ng, l, i, a\}$*. Consider a homomorphism* $\Sigma \rightarrow \{\mathsf{C}, \mathsf{V}\}$ *maps each consonant to C and each vowel to V.*



(b) *Under-generation of the conventional construction of the inverse homomorphic image: only bakbak-; never baktal-*

Inverse homomorphism?

- Consider $L = \{a^i b^j a^i b^j \mid i, j \geq 1\}$.
- An alphabetic homomorphism $h : \{0, 1, 2\} \rightarrow \{a, b\}^*$ with $h(0) = a$, $h(1) = a$ and $h(2) = b$.

Inverse homomorphism?

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- $h^{-1}(\mathsf{L}) = \{(0|1)^i 2^j (0|1)^i 2^j \mid i, j \geq 1\}$
  - The language $\{w2^j w2^j \mid w \in \{0, 1\}^*, j \geq 1\} \subset h^{-1}(\mathsf{L})$.
  - including 1202, 11020002, 1012201022.

Inverse homomorphism?

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FSBM:

- FSA: the incurred crossing dependencies: ☹
- the augmented copying mechanism: ☹
  - only *identical* copies
  - not general cases of symbol correspondence.

Closure properties: a summary

Operations	Closed or not
union	✓
concatenation	✓
Kleene star	✓
homomorphism	✓
intersection with regular languages	✓
inverse homomorphism	✗?

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Typology of reduplication

- current machinery: local reduplication with *two* adjacent, completely identical copies.

Typology of reduplication

- current machinery: local reduplication with *two* adjacent, completely identical copies.
- non-local copies ☹

  Chukchee absolutive singular

  'voice' quli quli-qul
- multiple copies ☹

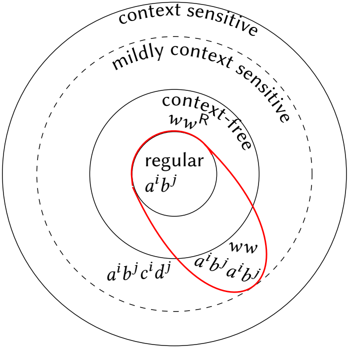
  Mokilese

  'give a shudder' roar roar-roar-roar *'continue to shudder'*
- non-identical copies ☹

  Javanese Habitual Repetitive

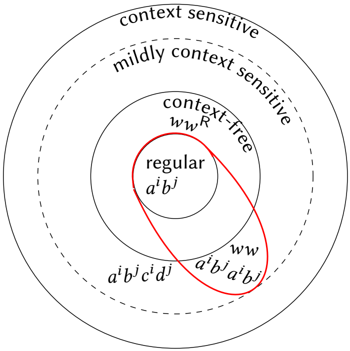
  'remember' eliŋ elaŋ-eliŋ

Conclusion



- **Finite-state buffered machines**
  - FSA + copying mechanism
  - compute productive total reduplication on any regular languages
- introduce a new class of languages incomparable to CFLs.
  - string reversal ✗ queue-like buffer
  - Swiss-German crossing dependencies ✗

Conclusion



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Thank you!

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Acknowledgments

I would like to express gratitude to Tim Hunter, Bruce Hayes, Dylan Bumford, Kie Zuraw, Ed Keenan, Claire Moore-Cantwell and the members of the UCLA Phonology Seminar for their comments, feedback and insights. Special thanks to the anonymous reviewers for their constructive suggestions and discussions. All errors remain my own.

Appendix: definition of homomorphism and inverse homomorphism

Definition

A (string) homomorphism is a function mapping one alphabet to strings of another alphabet, written $h : \Sigma \rightarrow \Delta^*$. We can extend h to operate on strings over Σ^* such that

1. $h(\epsilon_\Sigma) = \epsilon_\Delta$
2. for $w = a_1 a_2 \ldots a_n \in \Sigma^*$, $h(w) = h(a_1)h(a_2)\ldots h(a_n)$ where each $a_i \in \Sigma$

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An alphabetic homomorphism h_0 is a special homomorphism with h_0 maps each symbol in previous alphabet to another symbol in the new alphabet.

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Example: $h : \{\mathrm{H}, \mathrm{L}\} \to \{\mathrm{C}, \mathrm{V}\}^*$ with $h(\mathrm{H}) = \mathrm{CVC}$ and $h(\mathrm{L}) = \mathrm{CV}$.

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$h(\mathrm{LLH}) = h(\mathrm{L})h(\mathrm{L})h(\mathrm{H}) = \mathrm{CVCVCVC}$.

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$L_1 = \{(LH)^n \mid n \in \mathbb{N}\}$

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$$L_2 = \{(CV)^n(CVC)^n\} \to h^{-1}(L_2) = \{L^n H^n\}.$$

Appendix: An example run: ababb

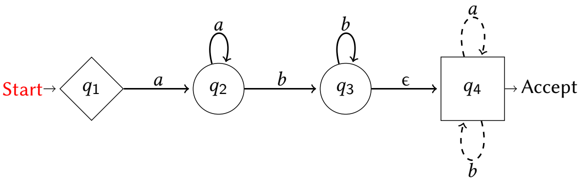

Figure: *An FSBM* M_2 *with* $G = \{q_1\}$ *and* $H = \{q_4\}$. $L(\mathsf{M}_2) = \{a^i b^j a^i b^j \mid i, j \geq 1\}$

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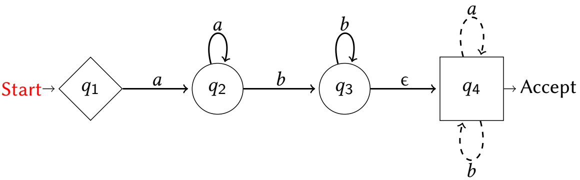

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input	state	buffer	mode
ababb	q_1	ϵ	N

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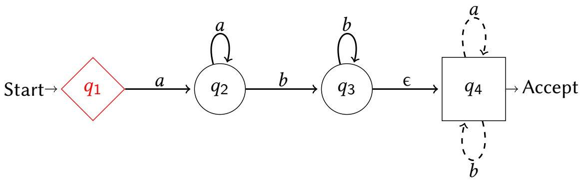

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ababb	q_1	ϵ	B

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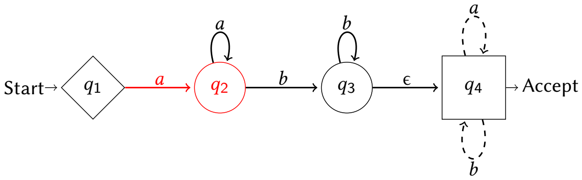

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babb	q_2	a	B

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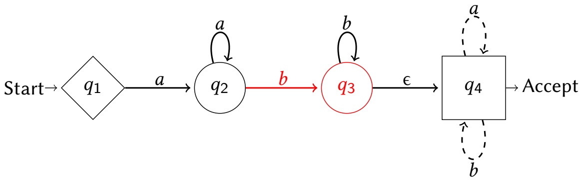

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abb	q_3	ab	B

Appendix: An example run: ababb

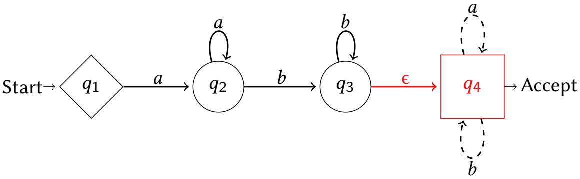

Figure: *An FSBM* M_2 *with* $G = \{q_1\}$ *and* $H = \{q_4\}$. $L(\mathsf{M}_2) = \{a^i b^j a^i b^j \mid i, j \geq 1\}$

input	state	buffer	mode
ababb	q_1	ϵ	N
ababb	q_1	ϵ	B
babb	q_2	a	B
abb	q_3	ab	B
abb	q_4	ab	B

Appendix: An example run: ababb

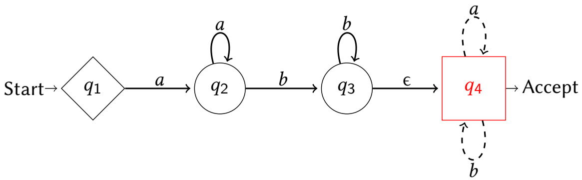

Figure: *An FSBM* M_2 *with* $G = \{q_1\}$ *and* $H = \{q_4\}$. $L(\mathsf{M}_2) = \{a^i b^j a^i b^j \mid i, j \geq 1\}$

input	state	buffer	mode
ababb	q_1	ϵ	N
ababb	q_1	ϵ	B
babb	q_2	a	B
abb	q_3	ab	B
abb	q_4	ab	B

input	state	buffer	mode
abb	q_4	ab	E

Appendix: An example run: ababb

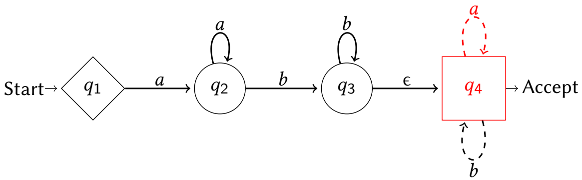

Figure: *An FSBM* M_2 *with* $G = \{q_1\}$ *and* $H = \{q_4\}$. $L(\mathsf{M}_2) = \{a^i b^j a^i b^j \mid i, j \geq 1\}$

input	state	buffer	mode
ababb	q_1	ϵ	N
ababb	q_1	ϵ	B
babb	q_2	a	B
abb	q_3	ab	B
abb	q_4	ab	B

input	state	buffer	mode
abb	q_4	ab	E
bb	q_4	b	E

Appendix: An example run: ababb

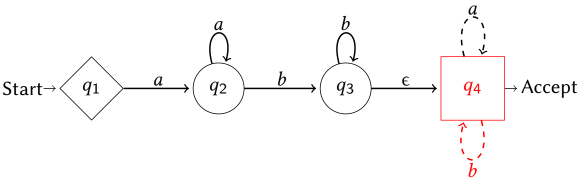

Figure: *An FSBM* M_2 *with* $G = \{q_1\}$ *and* $H = \{q_4\}$*.* $L(\mathsf{M}_2) = \{a^i b^j a^i b^j \mid i, j \geq 1\}$

input	state	buffer	mode
ababb	q_1	ϵ	N
ababb	q_1	ϵ	B
babb	q_2	a	B
abb	q_3	ab	B
abb	q_4	ab	B

input	state	buffer	mode
abb	q_4	ab	E
bb	q_4	b	E
b	q_4	ϵ	E

Appendix: An example run: ababb

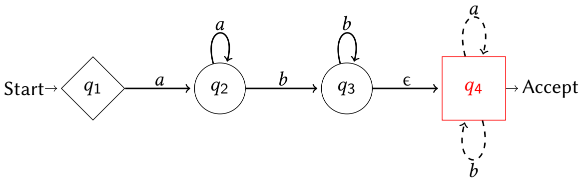

Figure: *An FSBM* M_2 *with* $G = \{q_1\}$ *and* $H = \{q_4\}$. $L(\mathsf{M}_2) = \{a^i b^j a^i b^j \mid i, j \geq 1\}$

input	state	buffer	mode
ababb	q_1	ϵ	N
ababb	q_1	ϵ	B
babb	q_2	a	B
abb	q_3	ab	B
abb	q_4	ab	B

input	state	buffer	mode
abb	q_4	ab	E
bb	q_4	b	E
b	q_4	ϵ	E
b	q_4	ϵ	N